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On "After 'Ohio Impromptu'" and the Use of Interval Resolutions[1]

1.0. Introduction

This paper will introduce the technique of interval resolution – a method for the generation of pitch and/or pitch class ratio sets in extended just intonation – in reference to the author's musical composition, "After 'Ohio Impromptu'", for two trombones (2009). Following this, suggestions for further compositional applications of interval resolution are made. Interval resolution is a derivative of reframing, a process described by the author in "Introduction to Reframing: intervallic augmentation and diminution of extended just intonation frequency ratios" (2011); however, the current presentation will not require previous knowledge of reframing. The paper does, however, assume competence in the use of frequency ratios as a compositional resource.

2.0. Interval Resolution

Interval resolution divides a given bounding interval (expressed as a frequency ratio, b, where $b > 1$)[2] into equal (arithmetic[3]) divisions. Following the pattern of ascending octaves in the harmonic

[1] "On 'After "Ohio Impromptu"'" was originally completed in December 2010 and was intended for an application that was abandoned shortly thereafter. It has been revised to its present form to complement "Introduction to Reframing," (Wadle 2011) with a description of an idiosyncratic adaptation of the process described therein. The piece, "After 'Ohio Impromptu'" (2009) was commissioned by trombonist Matthew Barbier in 2008. The project led to the formation of a performing duo with the author for the development of just intonation performance techniques and repertory for the trombone.

[2] Interval resolution is concerned with the division of a musical interval equal to the frequency ratio b, without reference to sequential presentation of the pitches defining that interval. This interval is to be expressed as a positive value that expresses the interval from the lower pitch ($^1/_1$) and b. Where some desired $b < 1$, b should be redefined as 1/1, and the upper note redefined as $^1/_b$, with $^1/_b$ serving as the bounding interval.

series, successive resolutions divide the bounding interval into 2^n parts, n being used to name the given resolution. So, the 2-resolution divides the bounding interval into $2^2 = 4$ equal parts. Of course, the concept can be adapted for divisions of the bounding interval in successive powers of some base other than two. For instance, if we wish to resolve some ratio into equal divisions of 3, according to interval resolution, we will have a first resolution of three parts, a 2-resolution of $3^2 = 9$ parts, a 3-resolution of $3^3 = 27$ parts, etc. To differentiate these alternative divisions, we can introduce a subscript to indicate bases other than two (e.g. the 3-resolution$_3$). We will restrain the present discussion to n-resolution$_2$s and omit the subscript.

Interval resolution may be applied to a stationary interval, moving through a sequence or resolutions, the higher resolutions generating more available pitches. Alternatively, as in "After 'Ohio Impromptu'", resolutions may be tied to the associated power of the bounding interval. So, if the bounding interval is $^5/_2$, the 2-resolution would begin at $(^5/_2)^2 = {}^{25}/_4$. To determine the succession of resulting frequency ratios in a given resolution, determine the difference, or span, between the bounding interval and the reference pitch ($^1/_1$); proceeding with our example: $^5/_2 - {}^1/_1 = {}^3/_2$. The span is then to be multiplied by 2^{-n} for the n-resolution, to provide the difference between successive terms of that resolution. We add this result, successively from $^1/_1$, $2^n - 1$ times to find the ratios for the given resolution. So, to find the 2-resolution of $^5/_2$, beginning at $^1/_1$, we multiply $2^{-2}(^3/_2) = {}^3/_8$, and then add successively to get the ratio set S:{$^1/_1$, $^{11}/_8$, $^7/_4$, $^{17}/_8$}. Where some other ratio is to be the starting point, multiply the additive term by that starting ratio and add the result, successively, to the starting ratio $2^n - 1$ times. Supposing we wish our resolution to begin at $^{25}/_4$, as suggested above, we then take $(^3/_8)(^{25}/_4) = {}^{75}/_{32}$ as the term to be added, successively, to $^{25}/_4$, resulting in a ratio set T:{$^{25}/_4$, $^{275}/_{32}$, $^{175}/_{16}$, $^{425}/_{32}$}.

[3] This arithmetic division leads to unequal perceived interval divisions owing to the geometric nature of pitch perception.

Alternatively, we can multiply the span between the bounding interval and the reference pitch by the successive ratios of the given resolution and then add one[4]. The successive ratios of the given resolution are determined by using the sequence of whole numbers to provide numerators over a denominator of 2^n, for all n where $0 \leq n < 1$. The members of the resulting set of frequency ratios are then multiplied by the desired starting ratio. Approaching our example in this way, we determine the span covered by the bounding interval as above: $^5/_2 - {}^1/_1 = {}^3/_2$. Then, we determine the set, R, of resolution ratios for the 2-resolution: R:{$^0/_4$, $^1/_4$, $^1/_2$, $^3/_4$}. We multiply span ratio, $^3/_2$, by each member of R and then add one for a resulting ratio set S:{$^1/_1$, $^{11}/_8$, $^7/_4$, $^{17}/_8$}. If we wish this ratio set to begin on some ratio other than the reference, we transpose the structure by multiplying all the ratios by that selected ratio. For instance, if the structure is to begin on $^{25}/_4$, we multiply each member of S by $^{25}/_4$, giving (once more) the set T:{$^{25}/_4$, $^{275}/_{32}$, $^{175}/_{16}$, $^{425}/_{32}$}.

3.0. "After 'Ohio Impromptu'": Selection of Target PCs

In the case of "After 'Ohio Impromptu'", resolutions one through three of the 13-limit primes – construed as both ascending bounding intervals (ratio values greater than one) and descending bounding intervals (ratio values less than one but greater than zero, which are the inverse of their positive counterparts[5]) – were calculated, anchoring each resolution to the corresponding power of that prime bounding interval (as discussed in section 2.0.). Once the ratio set of each resolution of these prime numbers (see Appendix 1) was determined, these sets were converted into pc-ratios (by multiplying or dividing by two until the ratio was between one and two: $1 \leq x^y \leq 2$, where y is an integer). A master list of pc-ratios, proper, in which the overtone and undertone generated sequences each had at least one ratio within 10¢ of the ratio of the other sequence, was then constructed. A series of thirty pc areas, some overlapping, was identified, these areas forming the

[4] We add one to return the ratios to their proper place in relation to the reference pitch ($^1/_1$).

[5] The resulting ratios for these descending bounding intervals are the inverses of the resulting ratios for the same resolution of the positive bounding interval of which the descending interval is, itself, the inverse. This allows us to circumvent the restriction on values b of bounding intervals introduced in section 2.0.

bases of the identification of a tolerance-adjusted compact set. This was accomplished by determining the sequence of frequency ratios within the thirty areas, including those returned by the resolution calculation along with others found within a lookup table (a greatly expanded form of Appendix 1 of Harry Partch's *Genesis of a Music*[6]), according to the principle of crystal growth (see Tenney 2008). The lookup ratios were bound by a window extending 10¢ below the highest ratio returned by the resolution calculations and 10¢ above the lowest (see Appendix 2). In other words, an unequal scale of thirty degrees was created, each degree having an acceptable margin of error in the vicinity of 10¢ (some less some more, none greater than 16¢). Within that margin of error a pc-ratio for precise tuning was identified that would lead to the simplest set of harmonic relations throughout the set of thirty pc's, weighting first for pc's expressed as frequency ratios in the octave above $^{1}/_{1}$, then for the same pc-ratios expressed as pitch ratios in the octave below $^{1}/_{1}$ (the inverse of the pc-ratio set). This algorithm defined, not only which pc-ratio was to be preferred, but also returned an ordering of them, from greatest to least overall harmonicity, using the order from the weighting for pc-ratios for the first trombone part, their inversions for the second trombone part. As we see in Appendix 3, the result is that the two voice's target pc's are related by interval class inversion, finally arriving at a unison on a nearly equal-tempered tritone, pc$^{140}/_{99}$.

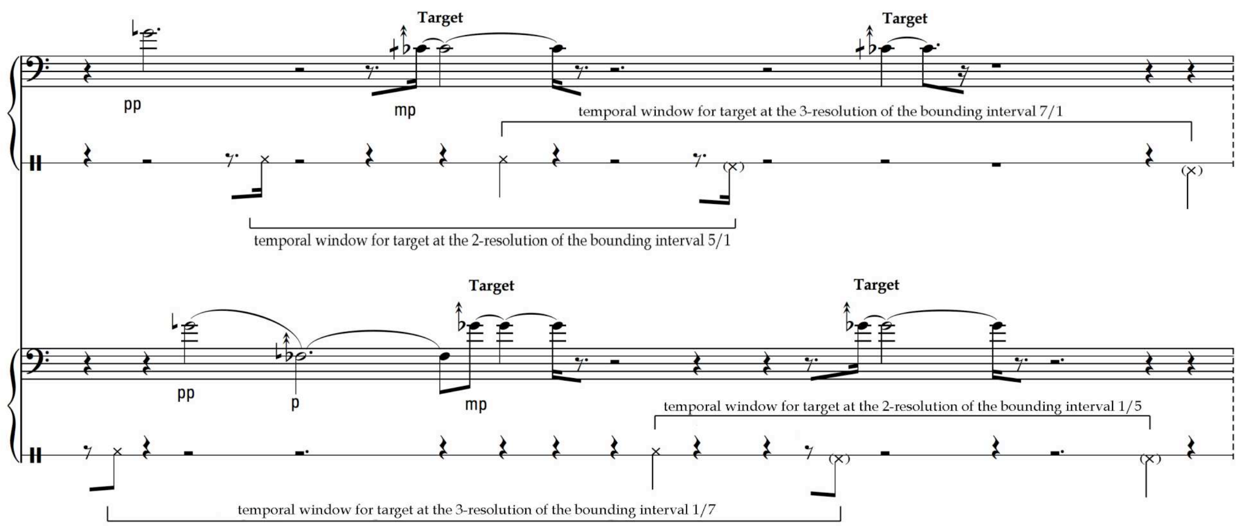
These tolerance-adjusted pc-ratios were treated as targets, to be tuned within an optimal octave for the trombones, A3 – A4. Proceeding without this octave restriction would have made the resulting piece impossibly rangy – at least for the author, who wrote the piece for himself (2nd trombone) and Matthew Barbier (1st trombone), aka ,, duo (Duo Comma). The origin of each pc-ratio as some resolution of some prime is indicated by a few counterbalancing expediencies including the aforementioned anchoring of the resolutions to the corresponding power of the bounding interval, the use of grace notes to indicate octave displacements where a bounding

[6] Since composing this piece, the author has generated a much more refined lookup table based upon Marc Sabat and Wolfgang von Schweinitz's list of tunable intervals (unpublished), which includes all pitches theoretically tunable in three steps, with the imposition of certain, rather liberal, constraints.

interval included frequency ratios some number of octaves apart, and – perhaps most noticeably – through the rhythmic and metrical treatment of the material.

3.1. "After 'Ohio Impromptu'": Temporal Placement of Target PCs

Temporal placement of notes within an eight-part division of a number of beats equal to twice the size of the bounding interval (prime numbers *n*, where $3 \leq n \leq 13$) was determined by a measurement of the 3-resolution resolution ratios' HDs from $^1/_1$. That is, all pc-ratios through the sequence of resolutions, that are generated by the same resolution ratio occupy the same position within the eight-part division of the temporal window (defined by the bounding interval) in which a target pc is to be tuned. Pcs derived from identical resolution ratios but different resolutions of a bounding interval (e.g. $^1/_2$ will appear in all resolutions greater than or equal to the 1-resolution) are (rhythmicaly) differentiated from one another by their durations: the pcs belonging to the 3-resolution divide the span into eight equal parts, those of the 2-resolution into four equal parts, those of the 1-resolution into two equal parts. These temporal windows are marked by the striking of the trombone bell, allowing it to ring, at their opening, and the striking of the bell so that it does not ring to mark its closing (see example 1). Note, in the example below, that multiple windows may be open in each voice. The piece begins with a simple sequential presentation of successive windows, moving towards increasing amounts of overlap between and within voices.

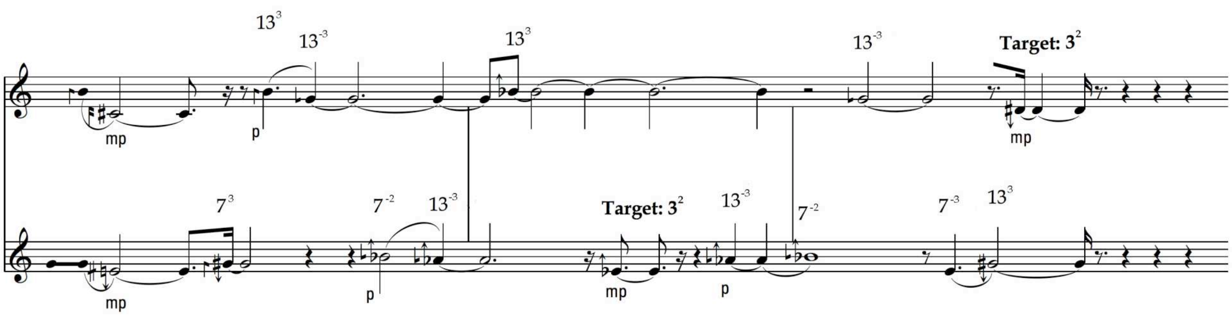


Example 1. *An illustration of temporal windows for target pc placement.*

In Example 1, overlapping temporal windows for target pc placement are found in both parts. The opening of the window coincides with the striking of the trombone bell so that it rings, indicated by an x-shaped notehead immediately above the single line staff of each part. The closing of the window is marked by the striking of the bell so that it does not ring, notated by an x-shaped notehead in parentheses immediately below the single line staff.

3.2. After 'Ohio Impromptu'": The Use of Open Resolutions

Of course, there is much more to the music than this particular sequence of pc ratios and their temporal placement – indeed, there are more pcs than this sequence would require. This is due to the introduction of an available set of tuning pitches drawn from the total set of available pcs in the thirty-degree scale. A restriction was imposed on this set: pc ratios from some particular resolution might be used to tune any target pc so long as one pitch from that resolution had already been tuned in the voice in question (the one in which the next target pitch was to be tuned) and so long as the last target pitch of that resolution of a prime had not yet been tuned. If these conditions were met, the particular resolution of the particular prime was considered "open", meaning its pcs (and tolerance variations thereon) were available for use as tuning pitches (see example 2).



Example 2. *An illustration of the use of open resolutions to provide tuning pitches.*

In Example 2, the bounding intervals are represented as a prime number to some power (negative powers indicate an inversion). The absolute value of the power of each bounding interval is equal to the resolution of the span defined by that bounding interval. This follows from the anchoring of the *n*-resolution to b^n, as discussed in section 2.0. It will be seen, then, that the 3-resolutions of the bounding intervals $^{7}/_{1}$, $^{13}/_{1}$, and $^{1}/_{13}$ and the 2-resolution of $^{1}/_{7}$ are used to tune the target pc E♭↑ ($^{64}/_{45}$), itself associated with the 2-resolution of $^{1}/_{3}$. Then, the 2 and 3-resolutions of $^{1}/_{7}$ and the 3-resolution of $^{1}/_{13}$ are used to tune the target pc D♯↓ ($^{45}/_{32}$). Notice that each voice was permitted access to the pcs derived from resolutions of both ascending and descending bounding intervals (positive and negative powers, respectively), though the target pcs of trombone I are all drawn from resolutions of ascending bounding intervals and those of trombone II from resolutions of descending bounding intervals.

3.3. After 'Ohio Impromptu'": Another Application of Tolerance

In composing out the material generated in the manner discussed above, the principle of substitutability of pc-ratios within the tolerance-area scale was maintained. In other words, where one particular pc-ratio from within the set of which the target pc-ratio was a member was preferred for the sake of tunability with reference to the available set of tuning pitches, a substitution of the preferred for the target pc-ratio was allowed. The two uses of tolerance within this piece can be thought of as follows: the use governing the designation of a harmonically compact pc-ratio set was based upon a conjecture as to how the ear, confronted with some set of pitch-classes associated with the thirty-degree scale would likely sort the material into a cognizable (or more nearly cognizable) structure. This, then, was used to determine the ideal sequence of targets. Following from this same principle, any pc within the proper tolerance-degree step of the thirty-step scale should elicit, in the given structure, a theoretical interpretation as the target identified within the first application of tolerance; it will stand as an instance of the categorical perceptual unit that is the particular step in the thirty-degree scale. The experience of rehearsing and performing the work bears this out. Locally, tunable relationships expressed as dyads are perceivable with great accuracy while, sequentially, pc-ratios belonging to the same step, even where in close temporal proximity, and though outside of the just noticeable difference of ≈ 5¢, are experienced as instance of the same unit within the scale.

4.0. Interval Resolution, Reframing, and Scale Generation

The reader familiar with "Reframing Harmonic Space, Part I" will doubtless recognize interval resolution as an application of basic reframing restricted to abstract ratios, a, such that $1 \leq a \leq 2$, where there is some resolution ratio $a - 1$ for each a. The set of abstract ratios is further restricted so as to derive from a set of resolution ratios that equally divides the span between the bounding interval, b, and one, resulting in a denominator (before reducing to simplest terms) for each resolution ratio equal to x^n for some resolution, n, and some term, x, which supplies the basic number of initial units into which the span is to be divided (this paper has been focused on resolutions based on $x = 2$). But the transposability of the ratio set resulting from interval resolution clearly distinguishes the procedure of interval resolution from reframing. This difference is the result of a differing conception underlying each process: whereas interval resolution is concerned only with the projection of x^n divisions of one instance of the (moveable) bounding interval, pure reframing is concerned with the projection of any interval onto some other interval (or "frame") –

even where the projected frequency interval is greater than the frame – as a means of unrestricted frequency ratio transformation.

Interval resolution grew out of an observation regarding the structure of the overtone series, construed as successive resolutions of the $^2/_1$ bounding interval. It is most useful when applied to the creation of a set of pitch-class ratios (after a fashion), owing to a deviation of the resolution ratios from their purely "reframed" counterparts[7]. The term "resolution", itself, suggests the restriction to a pitch-class derived approach. The octave ($^2/_1$) is replaced with the bounding interval, which is resolved into a certain number of parts. The whole structure may be transposed at various powers of the bounding interval as a theoretical octave, creating a ratio set that is cyclical at the bounding interval. A similar approach, not employing equal divisions of the bounding interval may be used as a rule for the creation of scales that are not cyclical at the octave but, rather, at the bounding interval. In discussing this approach with James Tenney, he made the further suggestion that (identical) divisions of bounding intervals that are octave complements ($b_1 \cdot b_2 = 2$, for all b where $1 < b < 2$) be alternated, creating a combined ratio set that is cyclical at the octave. An extension of this approach would allow cyclicality at whatever interval, c, results from the compounding of a regular alternating sequence of bounding intervals ($b_1 \cdot b_2 = c$, for all b where $1 < b$).

5.0. Conclusion

The foregoing introduces a means of generating pitch and interval material using simple mathematics to creating equal arithmetic divisions of any chosen musical interval expressed as a frequency ratio. The technique is derived from the structure of the harmonic series. It has been my desire to show the musical application of the technique with respect to "After 'Ohio Impromptu'", and to briefly suggest other compositional uses for interval resolution. I have said a great deal about the mechanics of "After 'Ohio Impromptu'" piece but have yet to say anything bearing an obvious

[7] See Wadle 2011, for a discussion of reframing and section 2.1. for a discussion of this deviation, specifically.

relation to the work's poetics. It is not my intention to interpret myself for others. However, I will direct attention to the prevalence of indiscernibles, near indiscernibles, and mirror images – particularly in light of the literary allusion in the work's title" "Ohio Impromptu" is a short play by Samuel Beckett featuring two actors who are to be identically clad and as near as possible to one another in appearance.

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Appendix 1. *Pitch Ratios Resulting from Resolution Calculations used in "After 'Ohio Impromptu'"*

Resulting Pitch Ratios at given bounding interval

	3/1	5/1	7/1	11/1	13/1
0	1/1	1/1	1/1	1/1	1/1
1	1/1	1/1	1/1	1/1	1/1
	2/1	3/1	4/1	6/1	7/1
2	1/1	1/1	1/1	1/1	1/1
	3/2	2/1	5/2	7/2	4/1
	2/1	3/1	4/1	6/1	7/1
	5/2	4/1	11/2	17/2	10/1
3	1/1	1/1	1/1	1/1	1/1
	5/4	3/2	7/4	9/4	5/2
	3/2	2/1	5/2	7/2	4/1
	7/4	5/2	13/4	19/4	11/2
	2/1	3/1	4/1	6/1	7/1
	9/4	7/2	19/4	29/4	17/2
	5/2	4/1	11/2	17/2	10/1
	11/4	9/2	25/4	39/4	23/2

Resulting Pitch Ratios at given bounding interval

	3/1	5/1	7/1	11/1	13/1
0	1/1	1/1	1/1	1/1	1/1
1	1/1	1/1	1/1	1/1	1/1
	1/2	1/3	1/4	1/6	1/7
2	1/1	1/1	1/1	1/1	1/1
	2/3	1/2	2/5	2/7	1/4
	1/2	1/3	1/4	1/6	1/7
	2/5	1/4	2/11	2/17	1/10
3	1/1	1/1	1/1	1/1	1/1
	4/5	2/3	4/7	4/9	2/5
	2/3	1/2	2/5	2/7	1/4
	4/7	2/5	4/13	4/19	2/11
	1/2	1/3	1/4	1/6	1/7
	4/9	2/7	4/19	4/29	2/17
	2/5	1/4	2/11	2/17	1/10
	4/11	2/9	4/25	4/39	2/23

Resulting Pitch Ratios, anchored to power of bounding interval = resolution

	3/1	5/1	7/1	11/1	13/1
0	1/1	1/1	1/1	1/1	1/1
1	3/1	5/1	7/1	11/1	13/1
	6/1	15/1	28/1	66/1	91/1
2	9/1	25/1	49/1	121/1	169/1
	27/2	50/1	245/2	847/2	676/1
	18/1	75/1	196/1	726/1	1183/1
	45/2	100/1	539/2	2057/2	1690/1
3	27/1	125/1	343/1	1331/1	2197/1
	135/4	375/2	2401/4	11979/4	10985/2
	81/2	250/1	1715/2	9317/2	8788/1
	189/4	625/2	4459/4	25289/4	24167/2
	54/1	375/1	1372/1	7986/1	15379/1
	243/4	875/2	6517/4	38599/4	37349/2
	135/2	500/1	3773/2	22627/2	21970/1
	297/4	1125/2	8575/4	51909/4	50531/2

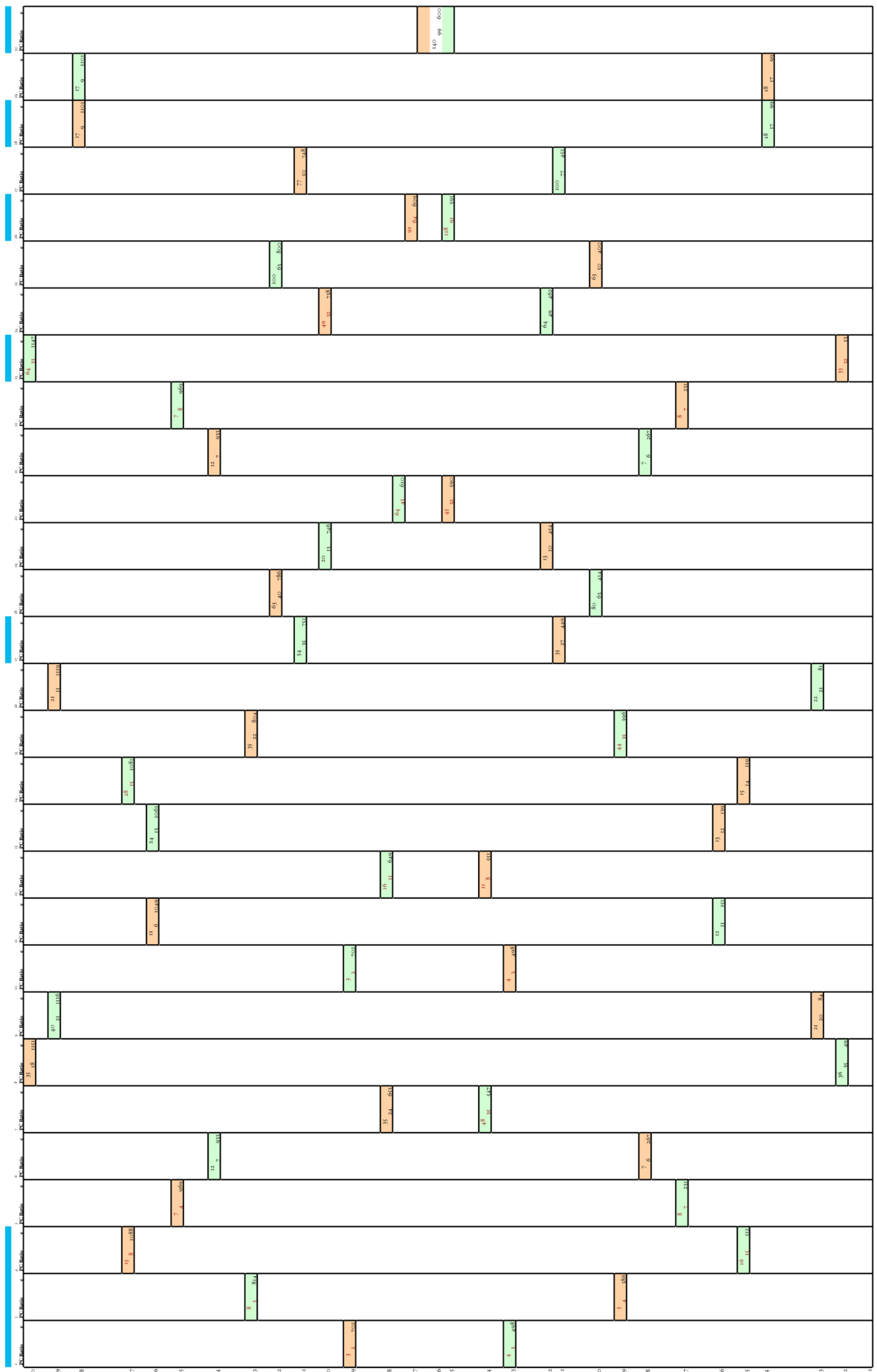
Resulting Pitch Ratios, anchored to power of bounding interval = resolution

	3/1	5/1	7/1	11/1	13/1
0	1/1	1/1	1/1	1/1	1/1
1	1/3	1/5	1/7	1/11	1/13
	1/6	1/15	1/28	1/66	1/91
2	1/9	1/25	1/49	1/121	1/169
	2/27	1/50	2/245	2/847	1/676
	1/18	1/75	1/196	1/726	1/1183
	2/45	1/100	2/539	2/2057	1/1690
3	1/27	1/125	1/343	1/1331	1/2197
	4/135	2/375	4/2401	4/11979	2/10985
	2/81	1/250	2/1715	2/9317	1/8788
	4/189	2/625	4/4459	4/25289	2/24167
	1/54	1/375	1/1372	1/7986	1/15379
	4/243	2/875	4/6517	4/38599	2/37349
	2/135	1/500	2/3773	2/22627	1/21970
	4/297	2/1125	4/8575	4/51909	2/50531

Appendix 2. *30-Degree Tolerance Area Scale with Associated PC Ratios.*

	pc ratio	pc ¢ value		pc ratio	pc ¢ value		pc ratio	pc ¢ value		pc ratio	pc ¢ value		pc ratio	pc ¢ value		pc ratio	pc ¢ value		pc ratio	pc ¢ value		pc ratio	pc ¢ value		pc ratio	pc ¢ value	
30	64/33	1146.7271	A	35/18	1151.2296	A	3993/2048	1155.9088	G																		
29	40/21	1115.5328	G	21/11	1119.463	A	16384/8575	1120.8949	G	245/128	1123.9655	A	5120/2673	1125.2208	A	48/25	1129.3276	A	121/63	1129.9	G						
28	17/9	1101.0454	A	189/100	1102.0635	A	121/64	1102.6359	G	256/135	1107.8213	A	243/128	1109.775	G	1024/539	1111.0302	G	154/81	1112.3238	A						
27	4096/2187	1078.417	A	28/15	1080.5572	A	144/77	1083.7662	G	4096/2187	1086.315	A	15/8	1088.2687	G												
26	11/6	1049.3629	G	90/49	1052.5719	F	8192/4459	1052.9946	F	147/80	1053.2931	A	81/44	1056.5021	G	3773/2048	1057.7957	A	448/243	1059.0509	A	24/13	1061.4273	G	1280/693	1062.2599	G
25	7/4	968.82591	G	65536/37349	973.46161	F	1280/729	974.58371	G	225/128	976.53743	F	16384/9317	977.22027	G	44/25	978.69051	G	640/363	981.72283	G	99/56	986.40204	F			
24	75/44	923.26449	F	4096/2401	924.69637	E	128/75	925.41757	G	2187/1280	927.37129	F	875/512	927.76705	F	77/45	929.92013	G	12/7	933.12909	F						
23	35/22	803.82168	F	6517/4096	803.99074	G	891/560	803.99833	E	8/5	813.68629	F															
22	63/40	786.42219	F	128/81	792.18	F	405/256	794.13372	E	8192/5175	795.18822	G	51909/32768	796.43649	E	192/121	799.31912	F	100/63	799.89152	E						
21	1120/729	743.40962	F	20/13	745.78605	E	2048/1331	746.04617	F	77/50	747.51642	F	50531/32768	749.85733	E	54/35	750.72538	E	25289/16384	751.46684	E	99/64	755.22794	E			
20	49/32	737.65181	F	135/88	740.86077	E	1120/729	743.40962	F	20/13	745.78605	E	2048/1331	746.04617	F	77/50	747.51642	F									
19	16384/10985	692.1033	E	112/75	694.24348	F	512/343	693.52228	D	121/81	694.81588	E	220/147	698.02484	D	433/289	699.95704	E	3/2	701.955	E						
18	16/11	648.68206	E	35/24	653.18462	E	11979/8192	657.86383	D																		
17	567/400	604.01848	E	363/256	604.59089	D	91/64	609.35357	E	64/45	609.77628	E	729/512	611.73001	D	77/54	614.27885	E									
16	512/363	595.40911	E	800/567	595.98152	D	140/99	599.91168	E	99/70	600.08832	D	567/400	604.01848	E	363/256	604.59089	D									
15	108/77	585.72115	D	1024/729	588.26999	E	45/32	590.22372	D	128/91	590.64643	D	512/363	595.40911	E												
14	16384/11979	542.13617	E	48/35	546.81538	D	11/8	551.31794	D																		
13	4/3	498.045	D	578/433	500.04296	D	147/110	501.97516	E	162/121	505.18412	D	75/56	505.75652	C	343/256	506.47772	E	10985/8192	507.8967	D						
12	100/77	452.48358	C	1331/1024	453.95383	C	13/10	454.21395	D	729/560	456.59038	C	176/135	459.13923	D	64/49	462.34819	C									
11	128/99	444.77206	D	32768/25280		C	35/27	449.27462	D	65536/50531	450.14267	D	100/77	452.48358	C	1331/1024	453.95383	C	13/10	454.21395	D	729/560	456.59038	C	176/135	459.13923	D
10	63/50	400.10848	D	121/96	400.68088	C	65536/51909	403.56351	D	5175/4096	404.81178	B	512/405	405.86628	D	81/64	407.82	C	80/63	413.57781	C						
9	5/4	386.31371	C	1120/891	396.00167	D	8192/6517	396.00926	B	44/35	396.17832	C															
8	7/6	266.87091	C	90/77	270.07987	B	1024/875	272.23295	C	2560/2187	272.62871	C	75/64	274.58243	B	2401/2048	275.30363	D	88/75	276.73551	C						
7	25/22	221.30949	B	9317/8192	222.77973	B	256/225	223.46257	C	729/640	225.41629	B	37349/32768	226.53839	C	8/7	231.17409	B									
6	13/12	138.57266	B	243/224	140.9491	A	4096/3773	142.20434	A	88/81	143.49794	B	160/147	146.7069	A	4459/4096	147.00538	C	49/45	147.4281	C	12/11	150.63706	B	35/32	155.13962	B
5	16/15	111.73129	B	2187/2048	113.68501	A	77/72	116.23385	B	15/14	119.44281	A	2197/2048	121.58299	B												
4	81/77	87.676155	A	539/512	88.969755	B	256/243	90.224996	B	135/128	92.178716	A	128/121	97.364115	B	200/189	97.936519	A	18/17	98.954592	A						
3	126/121	70.100023	B	25/24	70.672427	A	2673/2560	74.779233	A	256/245	76.034473	A	8575/8192	79.105147	B	22/21	80.537035	A	21/20	84.467193	B						
2	40/39	43.831051	A	4096/3993	44.091172	B	36/35	48.770381	A	33/32	53.272943	A															
1	1/1	0	A																								

Appendix 3. *Sequence of Target PC Ratios.*



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